Interpolation and oversampling in data converters. Trade-offs.

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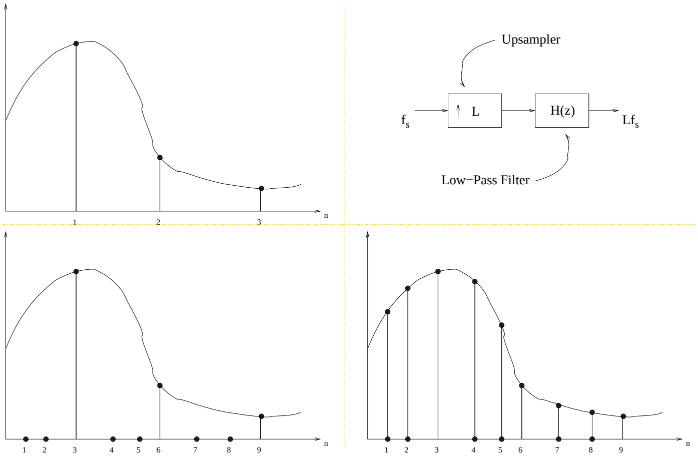
Upsampling and interpolation

- What is interpolation and why do we need it.



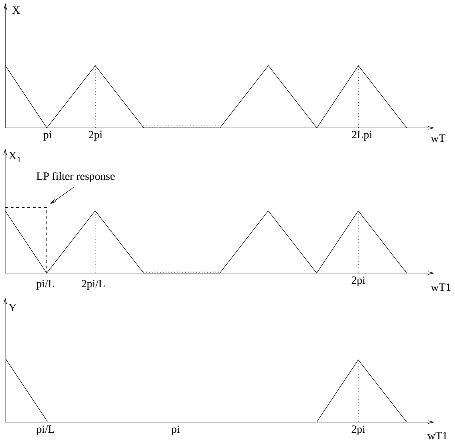
Basic principle of interpolation

Upsampling and interpolation

- The upsampled sequence, apart from the baseband, contains L-1 images of it over the spectrum
- These ideally should not be there
- That is what the LP filter does (the actual interpolation)
- The upsampler only insets zeros (to achieve the desired sampling frequency)

Upsampling and interpolation

- Spectrum of the three nodes of the interpolator (original, upsampled and interpolated).



Original, upsampled and interpolated sequences

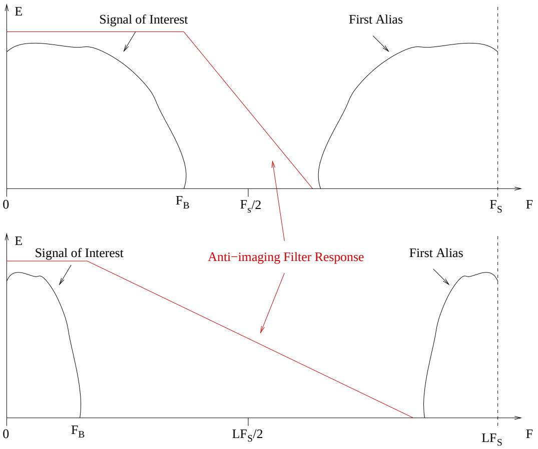
Upsampling and its effect with respect to aliasing

- However, in another case we need to go down to the original sampling rate as before
- The upsampled signal must be further filtered to suppress frequencies over $\frac{F_s}{2}$ (to avoid aliasing in the downsampling process)
- This however can also happen digitally together with the process of downsampling (discarding samples) in a so called decimation filter

Downsampling (decimation filter)

Upsampling and its effect with respect to aliasing

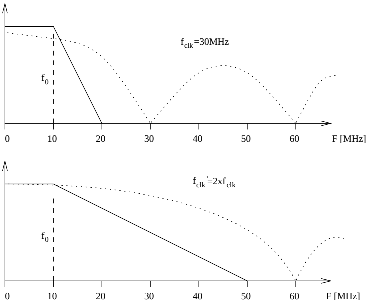
- One benefit of upsampling is that reduces the need of having precision time constant AA filter



Upsampling for AA filter precision reduction

Oversampling cont'd

- In terms of filter costs we can deduce that oversampling trades-off analog vs. digital circuitry
- A short example:



Analog filter requirements for different f_s

- Doubling f_s gives us a reduction of 10 to 5-6 pole filter (assuming 60dB attenuation and 6dB attenuation/pole)

Oversampling cont'd

For the case we mentioned:

OSR	FIR "firpmord" (Parks-McClellan)	Analog AI filter
2	23	$5 \sim 6$
4	37	$2 \sim 3$
8	57	$1 \sim 2$
16	103	1
32	195	1
64	381	1

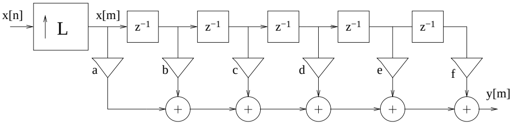
- One of the many imperical high-level power estimations for analog filters [1]:

$$P_a = 2 \times N \times \eta \times V_{dd}^2 \times f_{cut-off} \times DR^2 \quad (1)$$

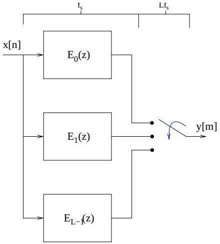
- and for direct form digital filter [2]:

$$P_d(N, coef, I, f) = (N \times (\gamma \times coef + \omega)) \times (I^{\alpha coef}) \times \frac{f}{f_0} \quad (2)$$

Oversampling cont'd



n	a	b	c	d	e	f	tap
0	0	.	.	.	.	.	y0
1	x	0	.	.	.	.	y1
2	x	x	0	.	.	.	y2
3	1	x	x	0	.	.	y3
4	x	1	x	x	0	.	y4
5	x	x	1	x	x	0	y5
6	2	x	x	1	x	x	y6
7	x	2	x	x	1	x	y7
8	x	x	2	x	x	1	y8



$$
\begin{aligned}
y_0 &= a_0\\
y_1 &= b_0\\
y_2 &= c_0\\
y_3 &= a_1 + d_0\\
y_4 &= b_1 + e_0\\
y_5 &= c_1 + f_0\\
y_6 &= a_2 + d_1\\
y_7 &= b_2 + e_1\\
y_8 &= c_2 + f_1
\end{aligned}
$$

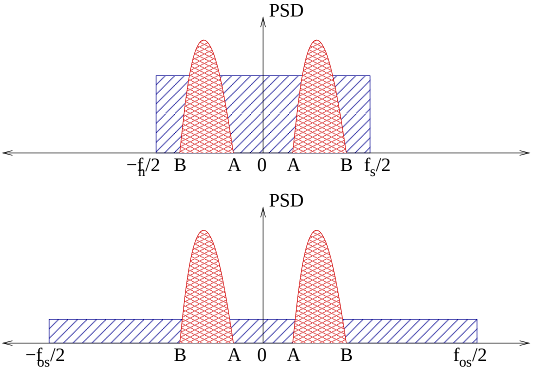
Oversampling cont'd

- More efficient IIR filters can be designed:
  - Butterworth, Cauer, Chebyshev
- Digital filters are also by far not ideal:
  - Filter coefficients precision (coefficient quantization)
  - Quantization of internal variables
  - Fixed vs floating point arithmetic
  - Rounding/truncation
  - Potential instability (IIR)
- To sum-up: the interpolation block by itself may not be easy to design

FIR FILTERS	IIR FILTERS
Easy to design	Require more effort
Always stable	May be unstable
Linear phase response	Non-linear phase response
Less efficient	More efficient

Oversampling cont'd

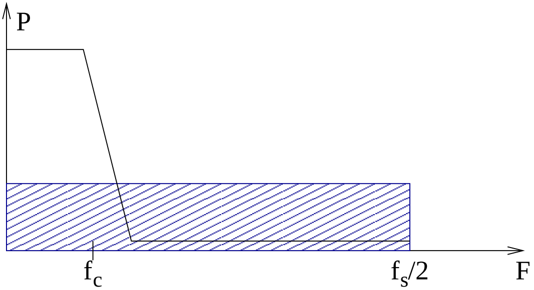
- Apart from the said above one of the main benefits of interpolation with oversampling is the spread of quantization noise over the band



Quantization noise spill throughout the OS range

Oversampling cont'd

- We can see that the in-band quantization noise will be much less after the decimation filter:



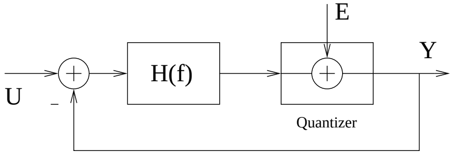
Quantization noise in-band and after filtering

- Theoretical SNR performance (straight oversampling)

$$SNR = 6.02N + 1.77 + 10log\left(\frac{f_{os}}{f_n}\right)[dB] \tag{3}$$

Noise shaping

- Oversampling with noise shaping



$\Sigma\Delta$ modulator (interpolator structure)

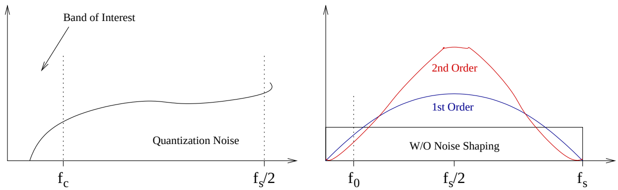
$$H = \frac{1}{f} \tag{4}$$

$$Y = \frac{1}{f}\left(U - Y\right) \tag{5}$$

$$Y = \frac{U}{f+1} + \frac{E.f}{f+1} \tag{6}$$

Noise shaping cont'd

- Effect on the noise spectrum:



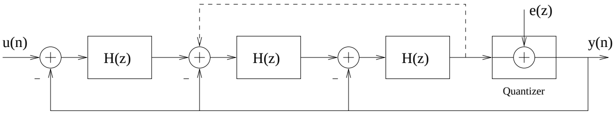
Quantization noise spectrum with noise shaping

- For a first order modulator

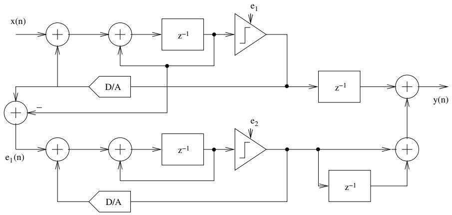
$$SNR_{max} = 6.02N + 1.76 - 5.17 + 30log(OSR) \qquad (7)$$

- Doubling OSR gives SNR improvement for 1'st order modulator of 9dB (no noise shaping gave us 3dB)

- High order modulators exist (two very popular):
- Interpolative Architecture



- Multi-Stage Noise Shaping



Multi-Stage Noise Shaping structure

Impacts of interpolation

- Increased dynamic performance SNR, SFDR, SINAD, etc. (quantization distortion is spread over the spectrum)
- Allows us to reduce the resolution of converters for the same performance requirements
- Trades-off filter complexity analog vs. digital
  - With analog filter order reduction, eventually noise induced from the filter is also decreased
  - Reduced sensitivity to process imperfections and drifts
- Choosing appropriate interpolation factor is a complex problem (with respect to filter design).

Impacts of interpolation

- Possible high complexity in digital filter design
- Possible large power reduction (case specific)
  - For high L, static and dynamic power in digital filters may become very significant
- Possible large area reduction (very case specific)
- Reduces the effects of sample jitter (every output sample is avg. over many input samples)